# AQA Physics A-level

## Topic 11: Engineering

### Key Points

# Moment of Inertia

The **moment of inertia** is a quantity that affects how an object or system reacts to an **angular acceleration**. For a single mass it is said that:

$$I = m r^2$$

*Where 'm' is the mass of the object and 'r' is its radius from the axis of rotation.*

If you are dealing with an extended object, you **sum** all the individual moments of inertia for all the particles that make up the body:

$$\Sigma I = m r^2$$

This could also be used to calculate the new moment of inertia if a **mass** is added to an object of known inertia 'I':

$$I_{new} = I + m r^2$$

# Rotational Kinetic Energy

The **rotational kinetic energy** of a body can be calculated using the equation:

$$E_k = \frac{1}{2} I \omega^2$$

*Where 'I' is the body's moment of inertia and 'ω' is its angular frequency.*

# Flywheels

A flywheel is a mechanical component that **accumulates** and **stores energy**. There are two main types, the first being flywheels that are used **exclusively** as an energy store, for example:

- **Regenerative braking in cars** use a flywheel to store the energy that would otherwise be wasted during braking - this energy is then used to aid vehicle acceleration at a later point
- **Push-and-Go cars** use a high-revving flywheel to store energy and then provide it to power the car's motion

The other main use of flywheels is to even out **fluctuations** in the rotational speed of a system.

# Energy Storage of a Flywheel

The **maximum** amount of energy that a flywheel can store is determined by two factors:

1. The wheel's **mass** and **shape**
2. The wheel's **maximum angular speed**

The maximum angular speed is itself determined by the **maximum breaking stress** of the material that the wheel is made from.

A mathematical expression can be formulated by considering the mass of a solid disc:

$$M = \pi R^2 t \rho$$

*Where 'M' is mass, 'R' is the radius, and 't' is the thickness.*

Substituting this into the expression for the moment of inertia of a solid disc gives:

$$I = \tfrac{1}{2} M R^2 \qquad I = \tfrac{1}{2} (\pi R^2 t \rho) R^2$$

Comparing this with the equation for rotational kinetic energy proves that $E_k$ is **proportional** to $R^4\omega^2$.

$$E_k = \tfrac{1}{2} I \omega^2 \qquad E_k = \tfrac{1}{4}\pi R^4 t \rho \omega^2$$

# Angular Quantities

You should be able to use the following equations for **rotational motion**:

$$\omega_2 = \omega_1 + \alpha t$$

$$\theta = \omega_1 t + \tfrac{1}{2}\alpha t^2$$

$$\omega_2^2 = \omega_1^2 + 2\alpha\theta$$

$$\theta = \tfrac{1}{2}(\omega_1 + \omega_2)t$$

It will also be useful to be familiar with the following **conversion**:

*1 revolution = 2π radians*

*1 rev $s^{-1}$ = 2π rad $s^{-1}$*

# Torque and Angular Acceleration

The equation linking **torque**, the **moment of inertia** and **angular acceleration** is:

$$T = I\alpha$$

The following points should be taken into account when using this equation:

- The unit for torque is **Nm**
- The torque used in calculation is the **resultant torque**
- **Frictional torques** often oppose the motion of a wheel and must be taken into account
- **Average** frictional torque can be calculated using the **deceleration** and **moment of inertia** of the system

# Angular Momentum

**Angular momentum** is equal to the product of the **moment of inertia** and the **angular frequency**. As an equation it is represented by:

$$Angular\ Momentum = I\omega$$

The unit for angular momentum is **Nms**.

Like with linear momentum, the angular momentum of a system is always conserved. The **law of conservation of angular momentum** states that:

*The angular momentum before and after an event is always conserved, provided that no external torque acts.*

The conservation of angular momentum can be used to explain ideas such as why divers must tuck in tightly to perform multiple somersaults. It can also be applied to **clutches**:

*Angular momentum before enabling the clutch* $= I1\omega1 + I2\omega2$

*Angular momentum after enabling the clutch* $=(I1 + I2)\ \omega$

*This means that..*

$$I1\omega1 + I2\omega2 = (I1 + I2)\ \omega$$

*Which allows the common angular speed to be calculated.*

# First Law of Thermodynamics

The **first law of thermodynamics** is a form of the **conservation of energy**. It is expressed as:

$$Q = \Delta U + W$$

*Where 'Q' = energy supplied by heat transfer, $\Delta U$ = change in internal energy, W = work done*

Or as expressed in words:

*Energy added or removed by heat transfer = change in internal energy + work done on/by gas*

You must apply the following **sign conventions** when using this formula:

- If there is an **increase** in internal energy, the $\Delta U$ value must be **positive** - if there is a decrease, it must be a negative value
- If work is done **on** a gas, for example through compressen, the W value must be **negative** - if work is done **by** the gas, for example through expansion, it must be a **positive** value

# Systems

When considering **thermodynamic** problems, it is usual to refer to **systems**. Systems are **regions** that contain a quantity of vapour or gas. There are **two** types that you should understand:

1. An **open system** is one where the gas or vapour can flow through the region it is in, as well as being able to pass into and out of the region into the surroundings
2. A **closed system** is one where the gas or vapour remains in the region, although it is important to note that the boundary of the region and the surroundings does **not** have to be fixed

An example of an open system is a aerosol can and an example of a closed system is gas expanding under a piston.

In both open and closed systems **heat** and **work** can enter and leave the region the gas or vapour is in.

# Isothermal Changes

An **isothermal** change is a change in which the **temperature** remains **constant**. When the temperature remains constant, the **average kinetic energy** of the particles also remains the same, meaning that the **internal energy** is also constant. The consequence of this is:

$$\Delta U = 0 \quad \textit{and so...} \quad Q = W$$

Perfectly isothermal changes are in reality impossible, but theoretically could occur if the container was a perfect conductor and the change happened at such a slow rate that the gas remained at thermal equilibrium with its surroundings. However, processes can often be modelled as being isothermal.

Isothermal changes always follow Boyle's Law:

$$pV = constant \quad \textit{or more usefully...} \quad p_1V_1 = p_2V_2$$

# Adiabatic Changes

An **adiabatic change** is one where no heat can pass into or out of the gas. This means that if the gas does **work**, the energy must come from the **internal energy** of the gas. In an adiabatic process:

$Q = 0$ *and so...* $W = -\Delta U$

This means that when a gas does work on its surroundings, the **internal energy** of the particles will decrease. Consequently, the **average kinetic energy** of the particles will reduce, and so the **temperature** of the gas will decrease.

Adiabatic changes also obey the following:

$pV^{\gamma} = constant$ *or more usefully...* $p_1V_1^{\gamma} = p_2V_2^{\gamma}$

# Constant Pressure Changes

If a gas is held at **constant pressure** by a piston pressing down on it, and then is heated, the piston will move upwards. If we call the distance the piston moves upwards $\Delta x$ we can say that:

*Work Done = Force x* $\Delta x$ *where...* *Force = Pressure x Piston Area*

*Combining these expressions gives...*

$$\text{Work Done} = p\,A\,\Delta x$$

*And so...*

$$W = p\Delta V$$

This demonstrates that **heating** a gas held at **constant pressure** will cause the volume to be **increased**. This increase in volume will also mean that **external work** is done. Equally by cooling a gas, the temperature and volume will decrease.

# Constant Volume Changes

If a gas in enclosed in a fixed space, the **volume** is **fixed**. This means that when the gas is **heated**, both the **pressure** of the gas and the **temperature** of the gas will **increase**. An increase of temperature also means an increase in **internal energy**. Since there is no volume change, **no work** is done on or by the gas and so we can say that:

$W = 0$ *and so...* $Q = \Delta U$

A consequence of this increase of **internal energy** is that products such as **gas bottles** shouldn't be stored in hot places or under direct sunlight. If they are, they may get heated and the internal energy may increase to potentially hazardous levels.

# p-V Diagrams

**p-V diagrams** are useful ways to **visualise** thermodynamic changes. They are graphs of **pressure against volume** that have the following properties:

- An **arrow** points in the direction that the change is happening
- **Isotherms** can be added to p-V diagrams, to represent constant temperatures
- The **closer** an isotherm is to the origin, the **lower** the temperature is

p-V diagrams are particularly useful when considering **engine cycles**.

# Four-Stroke Engines

A **four-stroke engine** is a type of **internal combustion engine**. This means that it involves **cylinders** that are filled with **air** and trapped by **pistons** that move up and down. Each piston movement either up or down is known as a **stroke**. Consequently, four-stroke engines are ones that **burn fuel** every **four strokes**. You need to be familiar with each stroke:

1. **Induction**: The piston moves outwards and so the volume of gas in the cylinder increases. Air and petrol is drawn into the cylinder through the open inlet valve. The pressure remains at a constant level, just below atmospheric pressure.
2. **Compression**: The valves are closed and the piston moves inwards meaning work is done to compress the gas. The volume decreases and the pressure increases.
3. **Expansion**
4. **Exhaust**

# Induction

- Piston moves outwards
- Volume of the contained area increases
- Air and petrol vapour is drawn in through the inlet valve
- Pressure remains constant
- The pressure level is just below atmospheric levels

# Compression

- Valves are closed
- The piston travels back inwards
- The volume decreases and the pressure increases
- Work is done since the gas is being compressed
- Towards the end of the stroke, the spark plug produces a spark
- The air/petrol mixture is ignited, causing a sudden increase in temperature and pressure, whilst the volume is constant

# Expansion

- Valves remain closed
- The increase in pressure forces the piston back out of the cylinder
- Work is done by the gas as it expands
- Towards the end of the outwards stroke, the exhaust valve opens
- The pressure returns to close to atmospheric levels

# Exhaust

- The piston moves back inwards
- The burnt fuel leaves through the exhaust valve
- The pressure remains at a constant level, just above atmospheric pressure levels

# Types of Efficiency

When considering mechanical systems such as engines, you need to be able to calculate the **system's efficiency**. Depending on what you are considering, there are several different efficiency equations that you need to be aware of:

$$\text{Mechanical Efficiency} = \frac{\text{Brake Power}}{\text{Indicated Power}}$$

$$\text{Thermal Efficiency} = \frac{\text{Indicated Power}}{\text{Input Power}}$$

$$\text{Overall Efficiency} = \frac{\text{Brake Power}}{\text{Input Power}}$$

You should also know how to calculate the **input power** of a fuel engine:

*Input Power = Calorific Value x Fuel Flow Rate*

Be aware that if the **calorific value** is in terms of **energy per mass**, the flow rate should be in **mass per second** as opposed to volume per second.

# Second Law of Thermodynamics

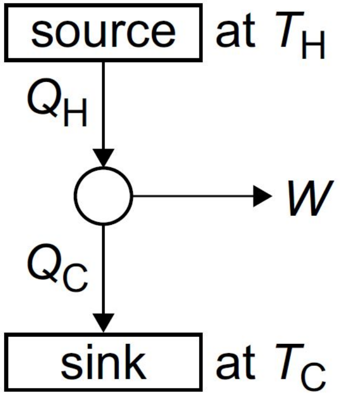
If a heat engine was only required to follow the first law of thermodynamics, all the **heat energy** supplied would be able to be transferred into **useful work**. In reality however, some of the energy is transferred to a **heat sink** that is at a **lower temperature** than the heat source, and so isn't transferred into useful work. This leads to the second law of thermodynamics, which states:

*"The efficiency of any thermodynamic process converting heat into work cannot approach 100%"*

If the temperature of a heat engine was to reach the same temperature as the heat source, it would be in **thermal equilibrium**. If this were to happen, **no heat** would flow between the two, and **no work** could be done. This demonstrates the necessity of the second law, and is why a heat engine must operate between the **heat source** and a **heat sink**.

# Efficiency of a Heat Engine

The **second law of thermodynamics** allows for an equation for the efficiency of a **heat engine** to be formulated:



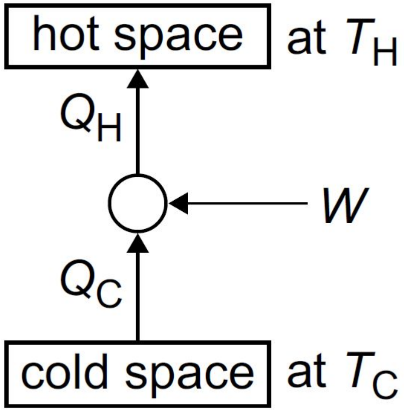

$$\textit{Efficiency} = \frac{\textit{Heat Transferred from Heat Source - Heat Transferred to Heat Sink}}{\textit{Heat Transferred from Heat Source}}$$

An equation for the **maximum theoretical efficiency** under **perfect conditions** can also be used:

$$\textit{Maximum Theoretical Efficiency} = \frac{\textit{Temperature of Heat Source - Temperature of Heat Sink}}{\textit{Temperature of Heat Source}}$$

# Reversed Heat Engines

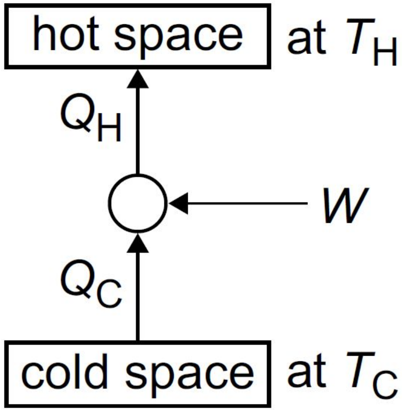
Much like a heat engine, a **reversed heat engine** also involves transfers between **hot** and **cold reservoirs**. Unlike for heat engines however, these reservoirs are simply known as hot and cold **spaces** rather than being referred to as sources and sinks. **Two** key examples of reversed heat engines that you need to be familiar with are:




1. Refrigerators
2. Heat Pumps

# Coefficient of Performance

Heat engines can be compared by a value known as their **coefficient of performance (COP)**. This quantity is a measure of the **quantity of work** that is converted into **heat transfer** by a given engine. How you calculate this coefficient depends on whether it is a refrigerator or heat pump system:




$$COP_{ref} = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C}$$

$$COP_{hp} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C} = \frac{T_H}{T_H - T_C}$$